

# SU(3) Flavor Dependence of the Heavy Quark Symmetry<sup>1</sup>

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## Abstract

SU(3) flavor dependence of the leading and subleading parameters appeared in the heavy quark expansion of the heavy-light mesons are systematically analyzed by using QCD sum rules.

## I Introduction

As the heavy quark goes into the infinite mass limit, the theory of strong interaction-QCD exhibits a new spin-flavor symmetry [1], i.e., the so-called heavy quark symmetry [1,2]. This new symmetry leads to many remarkable relations among the hadronic matrix elements between mesons with different spin and flavor, and reduced them to several independent universal functions[1,3,4]. These universal functions represent the nonperturbative dynamics of the weak decays of the heavy-light mesons. Therefore, to study them becomes very necessary, which would not only provide the clear magnitude of them and enlarge the predictive power of the heavy quark effective theory(HQET), but also would give us a better understanding of the nonperturbative nature of the strong interaction, especially would provide an improvement of our conventional nonperturbative approaches. In this talk, we present QCD sum rule

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analysis to both strange and non-strange heavy-light mesons to discuss the important property of the heavy quark symmetry—its SU(3) flavor dependence. Our investigation includes decay constants, masses as well as weak form factors.

## II. Heavy quark symmetry at the leading order

In HQET, the low energy parameter  $F_a(\mu)$  of heavy meson  $M_a(\bar{q}Q)$  is defined by [4]

$$\langle 0|\bar{q}\Gamma h_Q|M_a(v)\rangle = \frac{F_a(\mu)}{2}\text{Tr}[\Gamma M(v)], \quad (1)$$

where  $M(v)$  is the spin wavefunction of heavy meson  $M_a(v)$  in HQET

$$M(v) = \sqrt{m_Q}\frac{1+\not{v}}{2}(-i\gamma_5). \quad (2)$$

For convenient, from now on, we will omit all light flavor index  $a=u,d,s$  except in the numerical analysis.

Starting from the two-point correlation function in HQET,

$$\pi_5(\omega) = i \int d^4x e^{ik \cdot x} \langle 0|T A_5^{(v)}(x), A_5^{(v)+}(0)|0\rangle, \quad (3)$$

where  $A_5^{(v)} = \bar{q}\gamma_5 h_Q$  and  $\omega = 2k \cdot v$ , one obtains the sum rule for  $F(\mu)$ [5,6]

$$\begin{aligned} F^2(\mu)e^{-2\bar{\Lambda}/T} &= \frac{3}{8\pi^2} \int_{2m_q}^{\omega^c} ds \sqrt{s^2 - 4m_q^2} \{ [2m_q + s] \cdot [1 + \frac{\alpha_s(\mu)}{\pi}(2\ln \frac{\mu}{s} + \frac{4}{9}\pi^2 + \frac{17}{3})] \\ &+ m_q \frac{\alpha_s(\mu)}{\pi} [2\ln \frac{\mu}{s} + \frac{7}{3}] \} e^{-s/T} \\ &- \langle 0|\bar{q}q|0\rangle [1 - \frac{m_q}{2T} + \frac{m_q^2}{2T^2} + \frac{2\alpha_s(\mu)}{3\pi}] - \frac{\langle 0|\frac{\alpha_s}{\pi}GG|0\rangle m_q}{4T^2} [\gamma - 0.5 - \ln \frac{T}{\mu}] \\ &+ \frac{g_s \langle 0|\bar{q}\sigma Gq|0\rangle}{4T^2} + \frac{4\pi\alpha_s}{81T^3} \langle 0|\bar{q}q|0\rangle >^2 \end{aligned} \quad (4)$$

$$= J_F(\omega^c, T), \quad (5)$$

with  $\gamma = 0.5772$  being the Euler constant. Taking the derivative with respect to the inverse of  $T$ , one can obtain the sum rule for  $\bar{\Lambda}_a$ .

The Isgur-Wise function  $\xi(v \cdot v', \mu)$  is defined by the matrix element at the leading order in  $\frac{1}{m_Q}$  [7]

$$\langle M(v') | \bar{h}_{Q_2}(v') \Gamma h_{Q_1}(v) | M(v) \rangle = -\xi(v \cdot v', \mu) \text{Tr}[\bar{M}(v') \Gamma M(v)]. \quad (6)$$

Similar to the above approach, it is not difficult to get the sum rule for the Isgur-Wise function

$$\xi(y, \mu) = \frac{K(T, \omega^c, y)}{K(T, \omega^c, 1)}. \quad (7)$$

At the lowest order,  $K(T, \omega^c, y)$  is given by [5]

$$\begin{aligned} K(T, \omega^c, y) = & \frac{3}{8\pi^2} \left(\frac{2}{1+y}\right)^2 \int_{m_q}^{\omega^c} \sqrt{2(1+y)} d\alpha [\alpha + (1+y)m_q] \sqrt{\alpha^2 - 2(1+y)m_q^2} e^{-\alpha/T} \\ & - \langle 0 | \bar{q}q | 0 \rangle \left[ 1 - \frac{m_q}{2T} + \frac{m_q^2}{4T^2} (1+y) \right] \\ & + \langle 0 | \frac{\alpha_s}{\pi} GG | 0 \rangle \left[ \frac{y-1}{48T(1+y)} - \frac{m_q}{4T^2} (\gamma - 0.5 - \ln \frac{T}{\mu}) \right] \\ & + \frac{g_s \langle 0 | \bar{q}\sigma Gq | 0 \rangle}{4T^2} \frac{2y+1}{3} + \frac{4\pi\alpha_s \langle 0 | \bar{q}q | 0 \rangle^2}{81T^3} y. \end{aligned} \quad (8)$$

However, radiative correction in  $K(T, \omega^c, y)$  is very complex and will be presented in Ref.[8].

In the numerical analysis of sum rules, we take the parameters such as condensates and  $m_q$  as in [4-7] and set the scale  $\mu = 1\text{GeV}$ .

Evaluations of sum rules for  $F_a$  and  $\bar{\Lambda}_a$  give

$$\bar{\Lambda}_s \simeq 0.66 \pm 0.08\text{GeV}, \quad \hat{F}_s \simeq 0.48 \pm 0.08\text{GeV}^{3/2}, \quad (9)$$

$$\bar{\Lambda}_{u,d} \simeq 0.58 \pm 0.08\text{GeV}, \quad \hat{F}_{u,d} \simeq 0.39 \pm 0.07\text{GeV}^{3/2}. \quad (10)$$

where  $\hat{F}$  is a renormalization group invariant defined in Ref.[6].

Similarly, one also gets  $\Delta\bar{\Lambda} = \bar{\Lambda}_s - \bar{\Lambda}_{u,d}$  and the ratio  $R_F = F_s(\mu)/F_{u,d}(\mu)$  with the corresponding sum rules[6],

$$\Delta\bar{\Lambda} = 82 \pm 8\text{MeV}, \quad R_F = 1.23 \pm 0.03. \quad (11)$$

The numerical analysis [5] show that the Isgur-Wise function  $\xi_a(y)$  depends on the parameters  $\omega_a^c$  and T very weakly. At the center of the sum rule window  $T=0.8\text{GeV}$ , we find

the slope parameter  $\varrho_a^2$  defined as  $\varrho_a^2 = -\xi'_a(y=1, \mu)$  has an important property  $\varrho_s^2 > \varrho_{u,d}^2$  and  $R_{IW} = \xi_s/\xi_{u,d} \simeq (98.3 \pm 0.7)\%$  at  $y = 1.6$  (for  $\sigma(y) = 1$ ). Although different continuum model  $\sigma(y)$  gives different value for  $R_{IW}$ , the property  $R_{IW} < 1$  (for  $y \neq 1$ ) is independent of the model choice.

In summary, It is very interesting to find that the Isgur-Wise function for  $B_s \rightarrow D_s$  falls faster than the Isgur-Wise function for  $B_{u,d} \rightarrow D_{u,d}$ , which is just contrary to the prediction of the heavy meson chiral perturbation theory where only SU(3) breaking chiral loops are calculated [9]. Our result  $R_{IW} \leq 1$  agrees with that of other calculations [10]. It is expected that the future experiments can test this result and reveal the underlying mechanism of SU(3) breaking effects.

### III. Subleading corrections to the decay constants and the masses

In the HQET, the corresponding effective lagrangian is

$$L = \bar{h}_v i v \cdot D h_v + \frac{L_K}{2m_Q} + \frac{C_{mag}(\mu)L_S}{2m_Q}. \quad (12)$$

The two operators  $L_K$  and  $L_S$  have clear physical meaning:  $L_K$  is just the kinetic energy operator of the heavy quark

$$L_K = \bar{h}_v (iD)^2 h_v, \quad (13)$$

while  $L_S$  is the corresponding chromomagnetic interaction operator

$$L_S = \frac{1}{2} \bar{h}_v g_s \sigma_{\mu\nu} G^{\mu\nu} h_v, \quad (14)$$

whose Wilson coefficient  $C_{mag}(\mu)$  is given in a hybrid way [2]

$$C_{mag}(\mu) = Z^{-3/\beta_0} [1 + \frac{13}{6} \frac{\alpha_s}{\pi}] \quad (15)$$

with  $Z = \alpha_s(\mu)/\alpha_s(m_Q)$  and  $\beta_0 = 11 - \frac{2n_f}{3}$ . Then the decay constants can be expanded up to the  $\frac{1}{m_Q}$  order as

$$\langle 0 | \bar{q} \Gamma Q | M(v) \rangle = [C_1(\mu) + \frac{1+d_M}{4} C_2(\mu)] F(\mu) Tr[\Gamma M(v)] \{1 + \frac{1}{m_Q} [G_K(\mu) + a(\mu) \frac{m_q}{6} - \frac{\bar{\Lambda}}{6} b(\mu)]\}$$

$$+ \frac{2d_M}{m_Q} [C_{mag}(\mu)G_\Sigma(\mu) + A(\mu)\frac{m_q}{12} - \frac{\bar{\Lambda}}{12}B(\mu)]\}, \quad (16)$$

The  $\frac{1}{m_Q}$  corrections involve the three universal parameters:  $\bar{\Lambda}$ ,  $G_K$  and  $G_\Sigma$ . Among them,  $\bar{\Lambda}$  is the mass parameter, which measures the mass difference between the meson and heavy quark in the heavy quark limit. The universal parameters  $G_K$  and  $G_\Sigma$  come from insertions of the subleading operators in the effective lagrangian into the matrix element of the leading order current  $\bar{q}\Gamma h_v[4]$ , i.e.,

$$\langle 0 | i \int dx L_K(x), \bar{q}(0)\Gamma h_v(0) | M(v) \rangle = F(\mu)G_K(\mu)Tr[\Gamma M(v)], \quad (17)$$

$$\langle 0 | i \int dx L_\Sigma(x), \bar{q}\Gamma h_v(0) | M(v) \rangle = 2d_M F(\mu)G_\Sigma(\mu)Tr[\Gamma M(v)] \quad (18)$$

with  $d_P = 3$  for the pseudoscalar meson,  $d_V = -1$  for the vector meson.

The short distance coefficients  $C_1(\mu)$ ,  $C_2(\mu)$ ,  $b(\mu)$  and  $B(\mu)$  are given in [4,11]. For  $m_q \neq 0$ ,  $a(\mu)$  and  $A(\mu)$  are nonzero. As  $C_{mag}(\mu)$ , we express them in the hybrid approach in which the scale in the next -leading corrections is ambiguous for some unknown two-loop anomalous dimensions.

$$\begin{aligned} a(\mu) &= -2 + \frac{\alpha_s(m_Q) - \alpha_s(\mu)}{4\pi} \frac{S_{hl}}{2} - \frac{5\alpha_s(m_Q)}{6\pi} + x^{-4/\beta_0} \left[ 2 - \frac{\alpha_s(m_Q) - \alpha_s(\mu)}{2\pi} S_{hl} + \frac{7\alpha_s(m_Q)}{6\pi} \right] \\ &- x^{-3/\beta_0} \left[ \frac{13\alpha_s}{6\pi} - \frac{2\alpha_s(m_Q)}{\pi} + \frac{\alpha_s(m_Q) - \alpha_s(\mu)}{4\pi} S_{hl} \right] + \frac{5\alpha_s}{6\pi} x^{-2/\beta_0}, \end{aligned} \quad (19)$$

$$\begin{aligned} A(\mu) &= \frac{3}{8} S_{hl} \frac{\alpha_s(m_Q) - \alpha_s(\mu)}{\pi} - \frac{13\alpha_s(m_Q)}{6\pi} + \frac{7}{3} x^{-4/\beta_0} \left[ 1 - \frac{\alpha_s(m_Q) - \alpha_s(\mu)}{4\pi} S_{hl} + \frac{29\alpha_s(m_Q)}{42\pi} \right] \\ &- \frac{1}{3} x^{-3/\beta_0} \left[ 4 - \frac{\alpha_s(m_Q) - \alpha_s(\mu)}{\pi} \frac{5S_{hl}}{4} + \frac{14\alpha_s(m_Q)}{3\pi} - \frac{13\alpha_s}{6\pi} \right] - \frac{5\alpha_s}{18\pi} x^{-2/\beta_0}, \end{aligned} \quad (20)$$

$$\text{with } S_{hl} = 3 \frac{153-19n_f}{(33-2n_f)^2} - \frac{381+28\pi^2-30n_f}{36(33-2n_f)}.$$

In order to derive the sum rules for  $G_K$  and  $G_\Sigma$ , the correlation functions are chosen as [6,12]

$$\bar{\pi}_K(\omega) = i^2 \int dx dy e^{ik \cdot (x-y)} \langle 0 | T[\bar{q}\Gamma_M h_Q]_x, L_K(0), [\bar{h}_Q \bar{\Gamma}_M q]_y | 0 \rangle, \quad (21)$$

$$\bar{\pi}_\Sigma(\omega) = i^2 \int dx dy e^{ik \cdot (x-y)} \langle 0 | T[\bar{q}\Gamma_M h_Q]_x, L_\Sigma(0), [\bar{h}_Q \bar{\Gamma}_M q]_y | 0 \rangle. \quad (22)$$

By using the standard procedure, one can easily obtain the final sum rules

$$G_K(\mu) = \frac{1}{2} \frac{d}{dT} \left[ \frac{T J_K(\omega^c, T)}{J_F(\omega^c, T)} \right], \quad (23)$$

$$G_\Sigma(\mu) = \frac{1}{4} \frac{d}{dT} \left[ \frac{T J_\Sigma(\omega^c, T)}{J_F(\omega^c, T)} \right], \quad (24)$$

where  $J_K(\omega^c, T)$  and  $J_\Sigma(\omega^c, T)$  are evaluated up to two loops [6]. We find that the SU(3) breaking effects in the decay constant of the pseudoscalar are about 17% for the beauty meson and 13% for the charmed meson respectively, i.e.,  $f_{B_s}/f_B = 1.17 \pm 0.03$ ,  $f_{D_s}/f_D = 1.13 \pm 0.03$ , in which the SU(3) breaking effects in the subleading order is about  $-3\%$  of the corresponding leading one for the beauty meson, about  $-5.5\%$  for the charmed meson. In addition, the ratio of the vector to pseudoscalar meson decay constants are found to be  $f_{B_s^*}/f_{B_s} = f_{B^*}/f_B = 1.05 \pm 0.02$ ;  $f_{D_s^*}/f_{D_s} = 1.23 \pm 0.06$ ,  $f_{D^*}/f_D = 1.24 \pm 0.06$ .

Subleading corrections to the heavy meson masses include two parameters: the heavy quark kinetic energy parameter  $K$  and the chromomagnetic interaction parameter  $\Sigma$ , which are defined respectively by [13]

$$K = \langle M(v) | L_K | M(v) \rangle [\langle M(v) | \bar{h}_v h_v | M(v) \rangle]^{-1}. \quad (25)$$

and

$$\langle M(v) | L_\Sigma | M(v) \rangle = -d_M \Sigma(\mu) \langle M(v) | \bar{h}_v h_v | M(v) \rangle. \quad (26)$$

Luckily, the chromomagnetic interaction parameter  $\Sigma$  can be extracted from the experiment data and has been found to be almost independent of the light flavor [14]. However, it is significant to find from the QCD sum rules that the maximum SU(3) breaking effect in the heavy quark kinetic energy  $K$  is about 4% [15], i.e.,

$$K_{u,d} = -0.47 \pm 0.10 GeV^2, \quad K_s = -0.49 \pm 0.11 GeV^2 \quad (27)$$

and

$$R_K = K_s/K_{u,d} = 1.02 \pm 0.02. \quad (28)$$

## IV. Subleading Isgur-Wise functions

In order to get the subleading corrections to the weak form factors, we should construct the sum rules for the subleading Isgur-Wise functions  $\xi_{3,\chi_1,\chi_2}$  and  $\chi_3$ . Although the strange quark is more heavier than the up and down quarks, QCD sum rules analysis [16] show that the  $1/m_Q$  corrections to the weak form factors are small for all light flavors, especially the subleading Isgur-Wise functions  $\chi_{2,\chi_3}$  arisen by the chromomagnetic interactions are very small. The SU(3) breaking effects in all subleading Isgur-Wise functions are about  $18 \sim 20\%$  and almost independent of  $y$ .

## V. Summary

In this talk, we systematically discuss SU(3) flavor dependence of the heavy quark symmetry. To be specific, we evaluate decay constants of the heavy-light mesons, the fundamental mass observable  $\bar{\Lambda}$  in HQET, the heavy quark kinetic energy, Isgur-Wise function and subleading Isgur-Wise functions and their SU(3) breaking effects. We find i) the  $1/m_Q$  corrections to decay constants of the heavy-light mesons, especially for the charmed mesons are large, and for the SU(3) breaking quantities such as  $f_{B_s}/f_B, f_{D_s}/f_D$  etc., the  $1/m_Q$  corrections are about  $3\% \sim 7\%$ . ii) for SU(3) breaking effects in the mass, the main contribution is  $\Delta\bar{\Lambda}$ , and the heavy quark kinetic energy almost is flavor independent. iii) the Isgur-Wise function in HQET sum rules shows that SU(3) flavor symmetry is a good approximation but there is a discrepancy between result of HQET sum rules and that of the heavy meson chiral perturbation theory on the SU(3) breaking behavior. SU(3) breaking effects in all subleading Isgur-Wise functions are large and almost independent of  $y$ . iv) the  $1/m_Q$  corrections to the weak form factors are small, especially the subleading form factors  $\chi_2, \chi_3$  arisen by the chromomagnetic interactions are very small.

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